CHAPTER 4

SHOW ME THE MONEY: THE BASICS OF VALUATION

To invest wisely, you need to understand the principles of valuation. In this chapter, we examine those fundamental principles. In general, you can value an asset in one of three ways. You can estimate the intrinsic value of the asset by looking at its capacity to generate cashflows in the future. You can estimate a relative value, by examining how the market is pricing similar or comparable assets. Finally, you can value assets with cashflows that are contingent on the occurrence of a specific event as options.

With intrinsic valuation, we argue that the value of any asset is the present value of the expected cash flows on the asset, and it is determined by the magnitude of the cash flows, the expected growth rate in these cash flows and the uncertainty associated with receiving these cash flows. We begin by looking at assets with guaranteed cash flows over a finite period, and then we extend the discussion to cover the valuation of assets when there is uncertainty about expected cash flows. As a final step, we consider the valuation of a firm, with the potential, at least, for an infinite life and uncertainty in the cash flows.

With relative valuation, we begin by looking for similar or comparable assets. When valuing stocks, these are often defined as other companies in the same business. We then standardize convert the market values of these companies which are dollar values to multiples of some standard variable – earnings, book value and revenues are widely used. We then compare the valuations of the comparable companies to try to find misvalued companies.

There are some assets that cannot be valued using either discounted cashflow or relative valuation models because the cashflows are contingent on the occurrence of a specific event. These assets can be valued using option pricing models. We consider the basic principles that uuderlie these models in this chapter.

Intrinsic Value

We can estimate the value of an asset by taking the present value of the expected cash flows on that asset. Consequently, the value of any asset is a function of the cash flows generated by that asset, the life of the asset, the expected growth in the cash flows and the riskiness associated with the cash flows. We will begin this section by looking at valuing assets that have finite lives (at the end of which they cease to generate cash flows) and conclude by looking at the more difficult case of assets with infinite lives. We will also start the process by looking at firms whose cash flows are known with certainty and conclude by looking at how we can consider uncertainty in valuation.

The Mechanics of Present Value

Almost everything we do in intrinsic valuation rests on the concept of present value. The intuition of why a dollar today is worth more than a dollar a year from now is simple. Our preferences for current over future consumption, the effect of inflation on the buying power of a dollar and uncertainty about whether we will receive the future dollar all play a role in determining how much of a discount we apply to the future dollar. In annualized terms, this discount is measured with a discount rate. It is worth, however, reviewing the basic mechanics of present value before we consider more complicated valuation questions.

In general, there are five types of cash flows that we will encounter in valuing any asset. You can have a single cash flow in the future, a set of equal cashflows each period for a number of periods (annuity), a set of equal cashflows each period forever (perpetuity), a set of cashflows growing at a constant rate and each period for a number of periods (growing annuity) and a cash flow that grows at a constant rate forever (growing perpetuity).

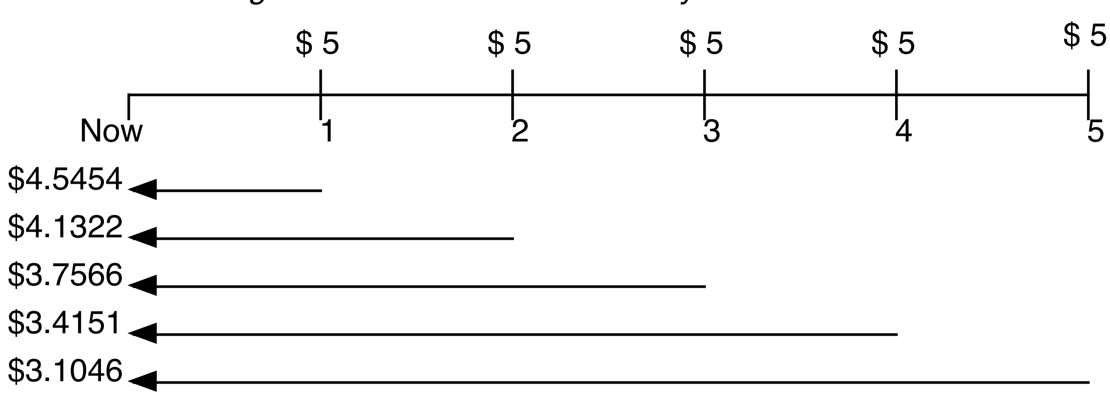
The present value of a single cashflow in the future can be obtained by discounting the cashflow back at the . Thus, the value of $ 10 million in 5 years, with a discount rate of 15% can be written as:

$$\text{Present value of \$ 10 million in 5 years} = \frac{\$10}{(1.15)^5} = \$\ 4.97 \text{ million}$$

You could read this present value to mean that you would be indifferent between receiving $4.97 million today or $ 10 million in 5 years.

What about the present value of an annuity? You have two choices. One is to discount each of the annual cashflows back to the present and add them all up. For instance, if you had an annuity of $ 5000 every year for the next 5 years and a discount rate of 10%, you could compute the present value of the annuity in figure 4.1:

Figure 4.1 Cash Flows on Annuiity



Adding up the present values yields \$18.95 million. Alternatively, you could use a short cut – an annuity formula – to arrive at the present value:

$$\text{PV of an Annuity} = A\left[\frac{1 - \frac{1}{(1+r)^n}}{r}\right] = 5\left[\frac{1 - \frac{1}{(1.1)^5}}{.10}\right] = \$18.95$$

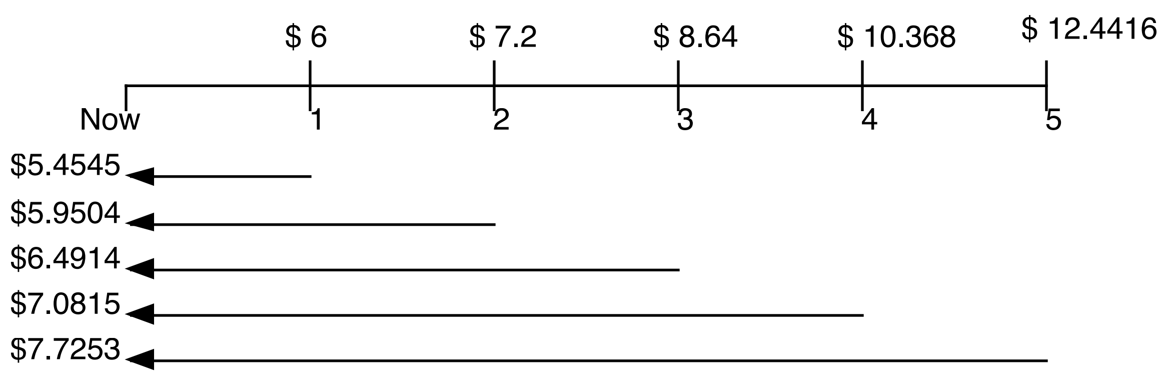
Getting from the present value of an annuity to the present value of a perpetuity is simple. Setting n to ∞ in the above equation yields the present value of a perpetuity

$$\text{PV of an Perpetuity} = A\left[\frac{1 - \frac{1}{(1+r)^\infty}}{r}\right] = \frac{A}{r}$$

Thus, the present value of \$ 5 million each year forever at a discount rate of 10% is \$ 50 million (\$5 million/ .10 = \$ 50 million)

Moving from a constant cashflow to one that grows at a constant rate yields a growing annuity. For instance, if we assume that the \$ 5 million in annual cashflows will grow 20% a year for the next 5 years, we can estimate the present value in figure 4.2:

Figure 4.2 Cash Flows on Growing Annuity



Summing up these present values yields a total value of \$32.70 million. Here again, there is a short cut available in the form of a growing annuity formula:

$$PV \text{ of a Growing Annuity} = A(1+g)\left[\frac{1 - \frac{(1+g)^n}{(1+r)^n}}{r - g}\right] = 5(1.20)\left[\frac{1 - \frac{(1.20)^5}{(1.10)^5}}{.10 - .20}\right] = \$32.70$$

Finally, consider a cashflow growing at a constant rate forever – a growing perpetuity. Substituting into the equation above, we get:

$$PV \text{ of a Growing Perpetuity } = A(1+g)\left[\frac{1 - \frac{(1+g)^{\infty}}{(1+r)^{\infty}}}{r - g}\right] = \frac{A(1+g)}{(r-g)}$$

Note that the fact the cashflows grow at a constant rate forever constrains this rate to be less than or equal to the growth rate of the economy in which you operate. Working with U.S. dollars, this growth rate should not exceed 5-6%.

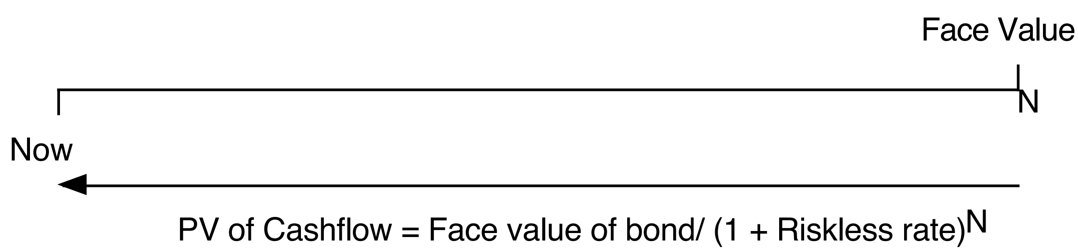
Valuing an Asset with Guaranteed Cash Flows

The simplest assets to value have cash flows that are guaranteed, i.e, assets whose promised cash flows are always delivered. Such assets are riskless, and the interest rate earned on them is called a **riskless rate**. The value of such an asset is the present value of the cash flows, discounted back at the riskless rate. Generally speaking, riskless investments are issued by governments that have the power to print money to meet any obligations they otherwise cannot cover. Not all government obligations are not riskless, though, since some governments have defaulted on promised obligations.

Default-free Zero-coupon Bond

The simplest asset to value is a bond that pays no coupon but has a face value that is guaranteed at maturity; this bond is a *default-free zero coupon bond*. We can show the cash flow on this bond as in Figure 4.3.

Figure 4.3: Cash Flows on N-year Zero Coupon Bond



The value of this bond can be written as the present value of a single cash flow discounted back at the riskless rate where N is the maturity of the zero-coupon bond. Since the cash flow on this bond is fixed, the value of the bond will increase as the riskless rate decreases and decrease as the riskless rate increases.

To see an example of this valuation at work, assume that the ten-year interest rate on riskless investments is 4.55%, and that you are pricing a zero-coupon treasury bond, with a maturity of ten years and a face value of $ 1000. The price of the bond can be estimated as follows:

$$\text{Price of the Bond} = \frac{\$1{,}000}{(1.0455)^{10}} = \$\ 640.85$$

Note that the face value is the only cash flow, and that this bond will be priced well below the face value of $ 1,000. Such a bond is said to be trading below par.

Conversely, we could estimate a default-free interest rate from the price of a zero-coupon treasury bond. For instance, if the 10-year zero coupon treasury were trading at $ 593.82, the default-free ten-year spot rate can be estimated as follows:

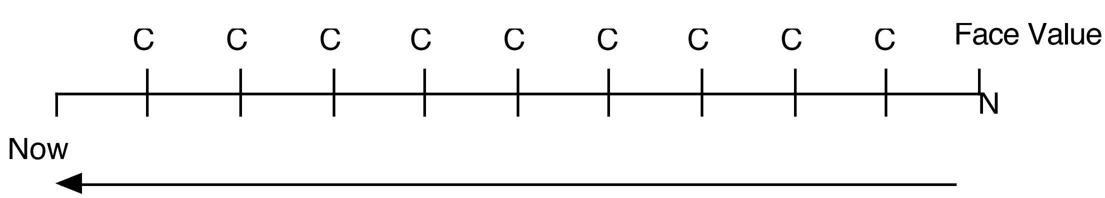
$$\text{Default-free Spot Rate} = \left(\frac{\text{Face Value of Bond}}{\text{Market Value of Bond}}\right)^{1/t} - 1 = \left(\frac{1000}{593.82}\right)^{1/10} - 1 = .0535$$

The ten-year default free rate is 5.35%.

Default-free Coupon Bond

Consider, now, a default-free coupon bond, which has fixed cash flows (coupons) that occur at regular intervals (usually semi annually) and a final cash flow (face value) at maturity. The time line for this bond is shown in Figure 4.4 (with C representing the coupon each period and N being the maturity of the bond).

Figure 4.4: Cash Flows on N-year Coupon Bond



Present value of cashflows = Present value of coupons + Present value of Face Value

This bond can actually be viewed as a series of zero-coupon bonds, and each can be valued using the riskless rate that corresponds to when the cash flow comes due:

$$\text{Value of Bond} = \sum_{t=1}^{t=N} \frac{\text{Coupon}}{(1+r_1)^1} + \frac{\text{Coupon}}{(1+r_2)^2} + \frac{\text{Coupon}}{(1+r_3)^3} \ldots + \frac{\text{Coupon}}{(1+r_N)^N} + \frac{\text{Face Value of the Bond}}{(1+r_N)^N}$$

where r_t is the interest rate that corresponds to a t-period zero coupon bond and the bond has a life of N periods.

It is, of course, possible to arrive at the same value using *some weighted average* of the period-specific riskless rates used above; the weighting will depend upon how large each cash flow is and when it comes due. This weighted average rate is called the *yield to maturity*, and it can be used to value the same coupon bond:

$$\text{Value of Bond} = \sum_{t=1}^{t=N} \frac{\text{Coupon}}{(1+r)^1} + \frac{\text{Coupon}}{(1+r)^2} + \frac{\text{Coupon}}{(1+r)^3} ... + \frac{\text{Coupon}}{(1+r)^N} + \frac{\text{Face Value of the Bond}}{(1+r_N)^N}$$

where r is the yield to maturity on the bond. Like the zero-coupon bond, the default-free coupon bond should have a value that varies inversely with the yield to maturity. As we will see shortly, since the coupon bond has cash flows that occur earlier in time (the coupons) it should be less sensitive to a given change in interest rates than a zero-coupon bond with the same maturity.

Consider now a five-year treasury bond with a coupon rate of 5.50%, with coupons paid every 6 months. We will price this bond initially using default-free spot rates for each cash flow in Table 4.1.

Table 4.1: Value of 5-year default-free bond

Time	*Coupon*	*Default-free Rate*	*Present Value*
0.5	$ 27.50	4.15%	$ 26.95
1	$ 27.50	4.30%	$ 26.37
1.5	$ 27.50	4.43%	$ 25.77
2	$ 27.50	4.55%	$ 25.16
2.5	$ 27.50	4.65%	$ 24.55
3	$ 27.50	4.74%	$ 23.93
3.5	$ 27.50	4.82%	$ 23.32
4	$ 27.50	4.90%	$ 22.71
4.5	$ 27.50	4.97%	$ 22.11
5	$ 1,027.50	5.03%	$ 803.92
			$ 1,024.78

The default-free spot interest rates reflect the market interest rates for zero coupon bonds for each maturity. The bond price can be used to estimate a weighted-average interest rate for this bond:

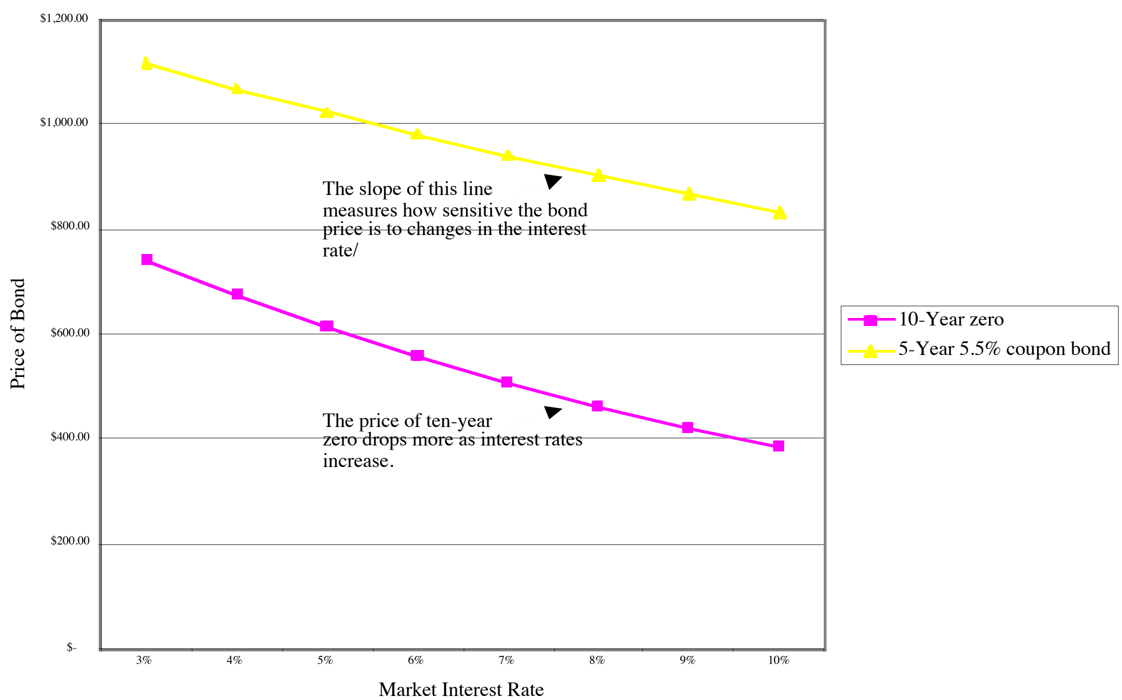
$$\$1{,}024.78 = \sum_{t=0.5}^{t=5} \frac{\$27.50}{(1+r)^t} + \frac{\$1{,}000}{(1+r)^5}$$

Solving for r, we obtain a rate of 4.99%, which is the yield to maturity on this bond.

Bond Value and Interest Rate Sensitivity and Duration

As market interest rates change, the market value of a bond will change. Consider, for instance, the 10-year zero coupon bond and the 5-year coupon bond described in the last two illustrations. Figure 4.5 shows the market value of each of these bonds as market interest rates vary from 3% to 10%.

Figure 4.5: Interest Rates and Bond Prices



> *bondval.xls*: See the spreadsheet that includes the bond valuation examples in this chapter.

Note that the price of the 10-year zero-coupon bond is much more sensitive to interest rate changes than is the 5-year coupon bond to a given change in market interest rates. The 10-year zero coupon bond loses about half its value as interest rates increase from 3% to 10%; in contrast, the 5-year 5.5% coupon bond loses about 30% of its value. This should not be surprising since the present value effect of that interest rate increases the larger the cash flow, and the further in the future it occurs. Thus longer-term bonds will be more sensitive to interest rate changes than shorter-term bonds, with similar coupons. Furthermore, low-coupon or no-coupon bonds will be more sensitive to interest rate changes than high-coupon bonds.

The interest rate sensitivity of a bond, which is a function of both the coupon rate and the maturity of the bond, can be captured in one measure called the <u>duration</u>. The greater the duration of a bond, the more sensitive its price is to interest rate movements.. The simplest measure of duration, called Macaulay duration, can be viewed as a weighted maturity of the different cash flows on the bond.

$$\text{Duration of a Bond} = \frac{\sum_{t=1}^{t=N} t \frac{CF_t}{(1+r)^t}}{\sum_{t=1}^{t=N} \frac{CF_t}{(1+r)^t}}$$

where r is the yield to maturity on the bond.

For a zero-coupon bond, which has only one cash flow, due at maturity, the duration is equal to the maturity.

Duration of 10-year zero-coupon bond = 10 years

The duration of the 5-year coupon bond requires a few more calculations, is calculated in the Table 4.2:

Table 4.2: Value of a 5-year Coupon Bond

Time (t)	Coupon	Present Value (at 4.99%)	t *Present Value
0.5	$27.50	$26.84	$13.42
1	$27.50	$26.19	$26.19
1.5	$27.50	$25.56	$38.34
2	$27.50	$24.95	$49.90
2.5	$27.50	$24.35	$60.87
3	$27.50	$23.76	$71.29
3.5	$27.50	$23.19	$81.17
4	$27.50	$22.63	$90.53
4.5	$27.50	$22.09	$99.40
5	$1,027.50	$805.46	$4,027.28
Sum		$1,025.02	$4,558.39

Duration of 5-year 5.5% coupon bond = $4,558/$1,025 = 4.45

The longer the duration of a bond, the more sensitive it is to interest rate changes. In our illustrations above, the ten-year coupon bond has a higher duration and will therefore be more sensitive to interest rate changes than the five-year coupon bond.

Introducing Uncertainty into Valuation

We have to grapple with two different types of uncertainty in valuation. The first arises in the context of securities like bonds, where there is a promised cash flow to the holder of the bonds in future periods. The risk that these cash flows will not be delivered is called **default risk**; the greater the default risk in a bond, given its cash flows, the less valuable the bond will become.

The second type of risk is more complicated. When we make equity investments in assets, we are generally not promised a fixed cash flow but are entitled, instead, to whatever cash flows are left over after other claim holders (like debt) are paid; these cash flows are called *residual cash flows*. Here, the uncertainty revolves around what these residual cash flows will be, relative to expectations. In contrast to default risk, where the risk can only result in negative consequences (the cash flows delivered will be less than promised), uncertainty in the context of equity investments can cut both ways. The actual cash flows can be much lower than expected, but they can also be much higher. For the moment, we will label this risk **equity risk** and consider, at least in general terms, how best to deal with it in the context of valuing an equity investment.

Valuing an Asset with Default Risk

We will begin a section on how we assess default risk and adjust interest rates for default risk, and then consider how best to value assets with default risk.

Measuring Default Risk and Estimating Default-risk adjusted Rates

When valuing investments where the cash flows are promised, but there is a risk that they might not be delivered, it is no longer appropriate to use the riskless rate as the discount rate. The appropriate discount rate here will include the riskless rate and an appropriate premium for the default risk called a **default spread**. In chapter 3, we examined how default risk is assessed by ratings agencies and the magnitude of the default spread. It is worth noting that even in the absence of bond ratings, lenders still assess default risk and charge default spreads.

Valuing an Asset with Default Risk

The most common example of an asset with just default risk is a corporate bond, since even the largest, safest companies still have some risk of default. When valuing a corporate bond, we generally make two modifications to the bond valuation approach we developed earlier for a default-free bond. First, we will discount the coupons on the corporate bond, even though these no longer represent expected cash flows, but are instead promised cash flows[1]. Second, the discount rate used for a bond with default risk will be higher than that used for default-free bond. Furthermore, as the default risk increases, so will the discount rate used:

$$\text{Value of Corporate Coupon Bond} = \sum_{t=1}^{t=N} \frac{\text{Coupon}}{(1+\text{k}_\text{d})^t} + \frac{\text{Face Value of the Bond}}{(1+\text{k}_\text{d})^N}$$

[1] When you buy a corporate bond with a coupon rate of 8%, you are promised a payment of 8% of the face value of the bond each period, but the payment may be lower or non-existent, if the company defaults.

where k_d is the market interest rate given the default risk.

Consider, for instance a bond issued by Boeing with a coupon rate of 8.75%, maturing in 35 years. Based upon its default risk (measured by a bond rating assigned to Boeing by Standard and Poor's at the time of this analysis), the market interest rate on Boeing's debt is 0.5% higher than the treasury bond rate of 5.5% for default-free bonds of similar maturity. The price of the bond can be estimated as follows:

$$\text{Price of Boeing bond} = \sum_{t=0.5}^{t=35} \frac{43.875}{(1.06)^t} + \frac{1,000}{(1.06)^{35}} = \$1,404.25$$

The coupons were assumed to be semi-annual and the present value was estimated using the annuity equation. Note that the default risk on the bond is reflected in the interest rate used to discount the expected cash flows on the bond. If Boeing's default risk increases, the price of the bond will drop to reflect the higher market interest rate.

Valuing an Asset with Equity Risk

Having valued assets with guaranteed cash flows and those with only default risk, let us now consider the valuation of assets with equity risk. We will begin with the introduction to the way we estimate cash flows and consider equity risk in investments with equity risk, and then we look at how best to value these assets.

Measuring Cash Flows for an Asset with Equity Risk

Unlike the bonds that we have valued so far in this chapter, the cash flows on assets with equity risk are not promised cash flows. Instead, the valuation is based upon the *expected cash flows* on these assets over their lives. We will consider two basic questions: the first relates to how we measure these cash flows, and the second to how to come up with expectations for these cash flows.

To estimate cash flows on an asset with equity risk, let us first consider the perspective of the owner of the asset, i.e. the equity investor in the asset. Assume that the owner borrowed some of the funds needed to buy the asset. The cash flows to the owner will therefore be the cash flows generated by the asset after all expenses and taxes, and also after payments due on the debt. This cash flow, which is after debt payments, operating expenses and taxes, is called the **cash flow to equity investors**. There is also a broader definition of cash flow that we can use, where we look at not just the equity investor in the asset, but at the total cash flows generated by the asset for both the equity investor and the lender. This cash flow, which is before debt payments but after operating expenses and taxes, is called the **cash flow to the firm** (where the firm is considered to include both debt and equity investors).

Note that, since this is a risky asset, the cash flows are likely to vary across a broad range of outcomes, some good and some not so positive. To estimate the expected cash flow, we consider all possible outcomes in each period, weight them by their relative probabilities[2] and arrive at an expected cash flow for that period.

Measuring Equity Risk and Estimate Risk-Adjusted Discount Rates

When we analyzed bonds with default risk, we argued that the interest rate has to be adjusted to reflect the default risk. This default-risk adjusted interest rate can be considered the **cost of debt** to the investor or business borrowing the money. When analyzing investments with equity risk, we have to make an adjustment to the riskless rate to arrive at a discount rate, but the adjustment will be to reflect the equity risk rather than the default risk. Furthermore, since there is no longer a promised interest payment, we will term this rate a risk-adjusted discount rate rather than an interest rate. We label this adjusted discount rate the **cost of equity**.

A firm can be viewed as a collection of assets, financed partly with debt and partly with equity. The composite cost of financing, which comes from both debt and equity, is a weighted average of the costs of debt and equity, with the weights depending upon how much of each financing is used. This cost is labeled the **cost of capital**.

For instance, assume that Boeing has a cost of equity of 10.54% and a cost of debt of 3.58%. Assume also that it raised 80% of its financing from equity and 20% from debt. Its cost of capital would then be

$$\text{Cost of Capital} = 10.58\% \ (.80) + 3.58\% \ (.20) = 9.17\%$$

Thus, for Boeing, the cost of equity is 10.54% while the cost of capital is only 9.17%.

If the cash flows that we are discounting are cash flows to equity investors, as defined in the previous section, the appropriate discount rate is the cost of equity. If the cash flows are prior to debt payments and therefore to the firm, the appropriate discount rate is the cost of capital.

Valuing an Asset with Equity Risk and Finite Life

Most assets that firms acquire have finite lives. At the end of that life, the assets are assumed to lose their operating capacity, though they might still preserve some value. To illustrate, assume that you buy an apartment building and plan to rent the apartments out to earn income. The building will have a finite life, say 30 to 40 years, at the end of which it

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[2] Note that in many cases, though we might not explicitly state probabilities and outcomes, we are implicitly doing so, when we use expected cash flows.

will have to be torn down and a new building constructed, but the land will continue to have value even if this occurs.

This building can be valued using the cash flows that it will generate, prior to any debt payments, and discounting them at the composite cost of the financing used to buy the building, i.e. , the cost of capital. At the end of the expected life of the building, we estimate what the building (and the land it sits on) will be worth and discount this value back to the present, as well. In summary, the value of a finite life asset can be written as:

$$\text{Value of Finite - Life Asset} = \sum_{t=1}^{t=N} \frac{\text{E(Cash flow on Asset}_t)}{(1+k_c)^t} + \frac{\text{Value of Asset at End of Life}}{(1+k_c)^N}$$

where k_c is the cost of capital.

This entire analysis can also be done from your perspective as the sole equity investor in this building. In this case, the cash flows will be defined more narrowly as cash flows after debt payments, and the appropriate discount rate becomes the cost of equity. At the end of the building's life, we still look at how much it will be worth but consider only the cash that will be left over after any remaining debt is paid off. Thus, the value of the equity investment in an asset with a fixed life of N years, say an office building, can be written as follows:

$$\text{Value of Equity in Finite - Life Asset} = \sum_{t=1}^{t=N} \frac{\text{E(Cash Flow to Equity}_t)}{(1+k_e)^t} + \frac{\text{Value of Equity in Asset at End of Life}}{(1+k_e)^N}$$

where k_e is the rate of return that the equity investor in this asset would demand given the riskiness of the cash flows and the value of equity at the end of the asset's life is the value of the asset net of the debt outstanding on it. Can you extend the life of the building by reinvesting more in maintaining it? Possibly. If you choose this course of action, however, the life of the building will be longer, but the cash flows to equity and to the firm each period have to be reduced[3] by the amount of the reinvestment needed for maintenance.

To illustrate these principles, assume that you are trying to value a rental building for purchase. The building is assumed to have a finite life of 12 years and is expected to have cash flows *before debt payments* of $ 1 million, growing at 5% a year for the next 12 years. The real estate is also expected to have a value of $ 2.5 million at the end of the 12th year (called the salvage value). Based upon your costs of borrowing and the cost you attach to

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[3] By maintaining the building better, you might also be able to charge higher rents, which may provide an offsetting increase in the cash flows.

the equity you will have invested in the building, you estimate a cost of capital of 9.51%. The value of the building can be estimated in Table 4.4:

Table 4.4: Value of Rental Building

Year	Expected Cash Flows	Value at End	PV at 9.51%
1	$ 1,050,000		$ 958,817
2	$ 1,102,500		$ 919,329
3	$ 1,157,625		$ 881,468
4	$ 1,215,506		$ 845,166
5	$ 1,276,282		$ 810,359
6	$ 1,340,096		$ 776,986
7	$ 1,407,100		$ 744,987
8	$ 1,477,455		$ 714,306
9	$ 1,551,328		$ 684,888
10	$ 1,628,895		$ 656,682
11	$ 1,710,339		$ 629,638
12	$ 1,795,856	$ 2,500,000	$ 1,444,124
		Value of Store =	$ 10,066,749

Note that the cash flows over the next 12 years represent a growing annuity, and the present value could have been computed with a simple present value equation, as well.

$$\text{Value of Building} = \frac{1{,}000{,}000\ (1.05)(1 - \frac{(1.05)^{12}}{(1.0951)^{12}}}{(.0951 - .05)} + \frac{2{,}500{,}000}{(1.0951)^{12}} = \$10{,}066{,}749$$

This building has a value of $10.07 million to you.

Now, consider the equity investment in the rental building described above. Assume that the cash flows from the building after debt payments are expected will be $ 850,000 a year, growing at 5% a year for the next 12 years. In addition, assume that the salvage value of the building, after repaying remaining debt will be $ 1 million at the end of the 12th year. Finally, assume that your cost of equity is 9.78%. The value of equity in this building can be estimated as follows:

$$\text{Value of Equity in Building} = \frac{850{,}000\ (1.05)\left(1 - \frac{(1.05)^{12}}{(1.0978)^{12}}\right)}{(.0978 - .05)} + \frac{1{,}000{,}000}{(1.0978)^{12}} = \$8{,}053{,}999$$

Note that the value of equity in the building is also an increasing function of expected growth and the building's life, and a decreasing function of the cost of equity.

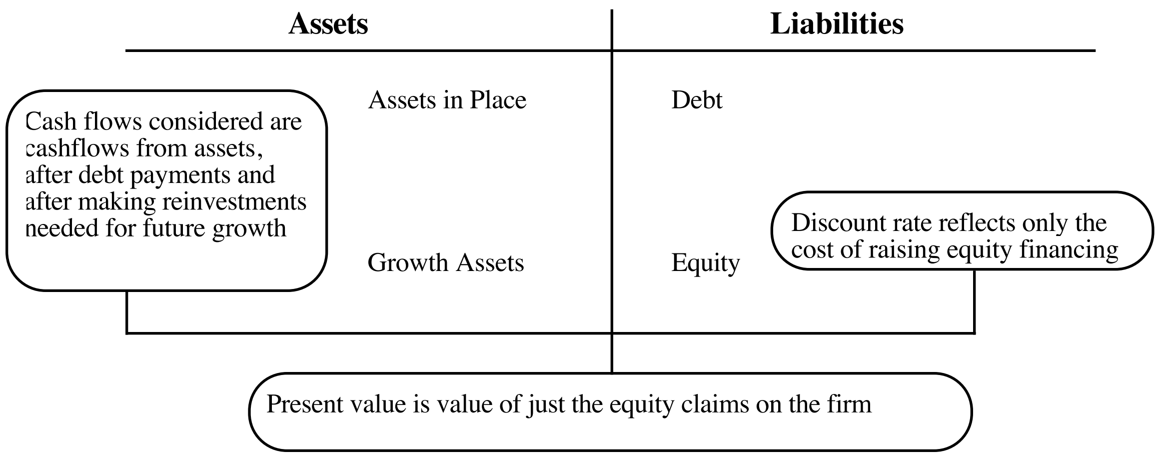
Valuing an Asset with an Infinite Life

When we value businesses and firms, as opposed to individual assets, we are often looking at entities that have no finite life. If they reinvest sufficient amounts in new assets each period, firms could keep generating cash flows forever. In this section, we value assets that have infinite lives and uncertain cash flows.

Equity and Firm Valuation

In the section on valuing assets with equity risk, we introduced the notions of cash flows to equity and cash flows to the firm. We argued that cash flows to equity are cash flows after debt payments, all expenses and reinvestment needs have been met. In the context of a business, we will use the same definition to measure the cash flows to its equity investors. These cash flows, when discounted back at the cost of equity for the business, yields the value of the equity in the business. This is illustrated in Figure 4.6:

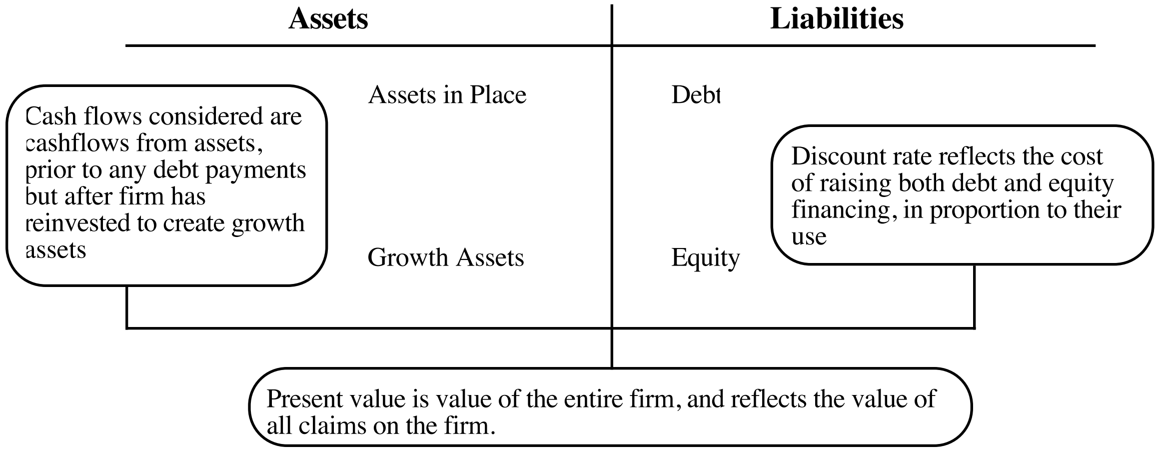
Figure 4.6: Equity Valuation



Note that our definition of both cash flows and discount rates is consistent – they are both defined in terms of the equity investor in the business.

There is an alternative approach in which, instead of valuing the equity stake in the asset or business, we look at the value of the entire business. To do this, we look at the collective cash flows not just to equity investors but also to lenders (or bondholders in the firm). The appropriate discount rate is the cost of capital, since it reflects both the cost of equity and the cost of debt. The process is illustrated in Figure 4.7.

Figure 4.7: Firm Valuation



Note again that we are defining both cash flows and discount rates consistently, to reflect the fact that we are valuing not just the equity portion of the investment but the investment itself.

Dividends and Equity Valuation

When valuing equity investments in publicly traded companies, we could argue that the only cash flows investors in these investments get from the firm are dividends. Therefore, the value of the equity in these investments can be computed as the present value of expected dividend payments on the equity.

$$\text{Value of Equity (Only Dividends)} = \sum_{t=1}^{t=\infty} \frac{E(\text{Dividend}_t)}{(1+k_e)^t}$$

The mechanics are similar to those involved in pricing a bond, with dividend payments replacing coupon payments, and the cost of equity replacing the interest rate on the bond. The fact that equity in a publicly traded firm has an infinite life, however, indicates that we cannot arrive at closure on the valuation without making additional assumptions.

One way in which we might be able to estimate the value of the equity in a firm is by assuming that the dividends, starting today, will grow at a constant rate forever. If we do that, we can estimate the value of the equity using the present value formula for a perpetually growing cash flow in chapter 3. In fact, the value of equity will be

$$\text{Value of Equity (Dividends growing at a constant rate forever)} = \frac{E(\text{Dividend next period})}{(k_e - g_n)}$$

This model, which is called the **Gordon growth model**, is simple but limited, since it can value only companies that pay dividends, and only if these dividends are expected to grow at a constant rate forever. The reason this is a restrictive assumption is that no asset or firm's cash flows can grow forever at a rate higher than the growth rate of the economy. If it did, the firm would become the economy. Therefore, the constant growth rate is constrained to be less than or equal to the economy's growth rate. For valuations of firms in US dollars, this puts an upper limit on the growth rate of approximately 5-6%[4]. This constraint will also ensure that the growth rate used in the model will be less than the discount rate.

We will illustrate this model using Consolidated Edison, the utility that produces power for much of New York city, paid dividends per share of $ 2.12 in 1998. The dividends are expected to grow 5% a year in the long term, and the company has a cost of equity of 9.40%. The value per share can be estimated as follows:

Value of Equity per share = $2.12 (1.05) / (.094 - .05) = $ 50.59

The stock was trading at $ 54 per share at the time of this valuation. We could argue that based upon this valuation, the stock was mildly overvalued.

What happens if we have to value a stock whose dividends are growing at 15% a year? The solution is simple. We value the stock in two parts. In the first part, we estimate the expected dividends each period for as long as the growth rate of this firm's dividends remains higher than the growth rate of the economy, and sum up the present value of the dividends. In the second part, we assume that the growth rate in dividends will drop to a stable or constant rate forever sometime in the future. Once we make this assumption, we can apply the Gordon growth model to estimate the present value of all dividends in stable growth. This present value is called the **terminal price** and represents the expected value of the stock in the future, when the firm becomes a stable growth firm. The present value of this terminal price is added to the present value of the dividends to obtain the value of the stock today.

$$\text{Value of Equity with high-growth dividends} = \sum_{t=1}^{t=N} \frac{\text{E(Dividends}_t)}{(1+k_e)^t} + \frac{\text{Terminal Price}_N}{(1+k_e)^N}$$

where N is the number of years of high growth and the terminal price is based upon the assumption of stable growth beyond year N.

$$\text{Terminal Price} = \frac{\text{E(Dividend}_{N+1})}{(k_e - g_n)}$$

To illustrate this model, assume that you were trying to value Coca Cola. The company paid $0.69 as dividends per share during 1998, and these dividends are expected

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[4] The nominal growth rate of the US economy through the nineties has been about 5%. The growth rate of the global economy, in nominal US dollar terms, has been about 6% over that period.

to grow 25% a year for the next 10 years. Beyond that, the expected growth rate is expected to be 6% a year forever. Assuming a cost of equity of 11% for Coca Cola, we can estimate the value of the stock in two parts and then estimate its value today.

I. Estimate the value of expected dividends during the next 10 years

The expected dividends during the high growth phase are estimated in the Table 4.5. The present values of the dividends are estimated using the cost of equity of 11% in the last column.

Table 4.5: Value of Expected Dividends during High-Growth Phase

Year	*Dividends per Share*	*Present Value*
1	$ 0.86	$ 0.78
2	$ 1.08	$ 0.88
3	$ 1.35	$ 0.99
4	$ 1.68	$ 1.11
5	$ 2.11	$ 1.25
6	$ 2.63	$ 1.41
7	$ 3.29	$ 1.58
8	$ 4.11	$ 1.78
9	$ 5.14	$ 2.01
10	$ 6.43	$ 2.26
	PV of Dividends	$ 14.05

II. Estimate the terminal value of the stock at the end of the high growth phase

To estimate the terminal price, we first estimate the dividends per share one year past the high growth phase and use the perpetual growth equation to compute present value. For Coca Cola, the estimates are as follows:

ddmginzu.xls: See the spreadsheet that contains the valuation of Coca Cola.

Expected Dividends per share in year 11 = $ 6.43 *1.06 = $ 6.81

Expected Terminal Price = $ 6.81 / (.11 - .06) = $ 136.24

III. Estimate the value of the stock today

To estimate the value of the stock today, we add the present value of the terminal price estimated in the previous step to the present value of the dividends during the high growth period:

Value of Stock today = PV of Dividends in high growth + PV of Terminal Price

$$= \$\ 14.05 \quad + \$\ 136.24/(1.11)^{10} = \$62.03$$

A Broader Measure of Cash Flows to Equity

There are two significant problems with the use of just dividends to value equity. The first is that it works only cash flows to the equity investors take the form of dividends.

It will not work for valuing equity in private businesses, where the owners often withdraw cash from the business but may not call it dividends, and it may not even work for publicly traded companies if they return cash to the equity investors by buying back stock, for instance. The second problem is that the use of dividends is based upon the assumption that firms pay out what they can afford to in dividends. When this is not true, the dividend discount models will mis-estimate the value of equity.

To counter this problem, we consider a broader definition of cash flow to which we call **free cash flow to equity**, defined as the cash left over after operating expenses, interest expenses, net debt payments and reinvestment needs. By **net debt payments**, we are referring to the difference between new debt issued and repayments of old debt. If the new debt issued exceeds debt repayments, the free cash flow to equity will be higher.

Free Cash Flow to Equity (FCFE) = Net Income – Reinvestment Needs – (Debt Repaid – New Debt Issued)

Think of this as potential dividends, or what the company could have paid out in dividend. To illustrate, in 1998, the Home Depot's free cash flow to equity using this definition was:

$FCFE_{Boeing}$ = Net Income – Reinvestment Needs – (Debt Repaid – New Debt Issued)

= $ 1,614 million - $1,876 million – (8 – 246 million) = - $ 24 million

Clearly, the Home Depot did not generate positive cash flows after reinvesment needs and net debt payments. Surprisingly, the firm did pay a dividend, albeit a small one. Any dividends paid by the Home Depot during 1998 had to be financed with existing cash balances, since the free cash flow to equity is negative.

fcfeginzu.xls: See the spreadsheet that contains the valuation of the Home Depot

Once the free cash flows to equity have been estimated, the process of estimating value parallels the dividend discount model. To value equity in a firm where the free cash flows to equity are growing at a constant rate forever, we use the present value equation to estimate the value of cash flows in perpetual growth:

$$\text{Value of Equity in Infinite - Life Asset} = \frac{E(FCFE_t)}{(k_e - g_n)}$$

All the constraints relating to the magnitude of the constant growth rate used that we discussed in the context of the dividend discount model, continue to apply here.

In the more general case, where free cash flows to equity are growing at a rate higher than the growth rate of the economy, the value of the equity can be estimated again in two parts. The first part is the present value of the free cash flows to equity during the high growth phase, and the second part is the present value of the terminal value of equity, estimated based on the assumption that the firm will reach stable growth sometime in the future.

$$\text{Value of Equity with high growth FCFE} = \sum_{t=1}^{t=N} \frac{E(FCFE_t)}{(1+k_e)^t} + \frac{\text{Terminal Value of Equity}_N}{(1+k_e)^N}$$

With the FCFE approach, we have the flexibility we need to value equity in any type of business or publicly traded company.

Consider the case of the Home Depot. Assume that we expect the free cash flows to equity at the firm to become positive next period and to grow for the next 10 years at rates much higher than the growth rate for the economy. To estimate the free cash flows to equity for the next 10 years, we make the following assumptions:

- The net income of $1,614 million will grow 15% a year each year for the next 10 years.
- The firm will reinvest 75% of the net income back into new investments each year, and its net debt issued each year will be 10% of the reinvestment.

Table 4.6 summarizes the free cash flows to equity at the firm for this period and computes the present value of these cash flows at the Home Depot's cost of equity of 9.78%.

Table 4.6: Value of FCFE

Year	Net Income	Reinvestment Needs	Net Debt Issued	FCFE	PV of FCFE
1	$ 1,856	$ 1,392	$ (139)	$ 603	$ 549
2	$ 2,135	$ 1,601	$ (160)	$ 694	$ 576
3	$ 2,455	$ 1,841	$ (184)	$ 798	$ 603
4	$ 2,823	$ 2,117	$ (212)	$ 917	$ 632
5	$ 3,246	$ 2,435	$ (243)	$ 1,055	$ 662
6	$ 3,733	$ 2,800	$ (280)	$ 1,213	$ 693
7	$ 4,293	$ 3,220	$ (322)	$ 1,395	$ 726
8	$ 4,937	$ 3,703	$ (370)	$ 1,605	$ 761
9	$ 5,678	$ 4,258	$ (426)	$ 1,845	$ 797
10	$ 6,530	$ 4,897	$ (490)	$ 2,122	$ 835
Sum of PV of FCFE =					$6,833

Note that since more debt is issued than paid, net debt issued increases the free cash flows to equity each year. To estimate the terminal price, we assume that net income will grow 6% a year forever after year 10. Since lower growth will require less reinvestment, we will assume that the reinvestment rate after year 10 will be 40% of net income; net debt issued will remain 10% of reinvestment.

$FCFE_{11}$ = Net $Income_{11}$ – $Reinvestment_{11}$ – Net Debt Paid $(Issued)_{11}$

= \$6,530 (1.06) – \$6,530 (1.06) (0.40) – (-277) = \$ 4,430 million

Terminal $Price_{10}$ = $FCFE_{11}/(k_e - g)$ = \$ 4,430 / (.0978 - .06) = \$117,186 million

The value per share today can be computed as the sum of the present values of the free cash flows to equity during the next 10 years and the present value of the terminal value at the end of the 10^{th} year.

Value of the Stock today = \$ 6,833 million + \$ 117,186/$(1.0978)^{10}$ = \$52,927 million

On a free cash flow to equity basis, we would value the equity at the Home Depot at \$ 52.93 billion.

From Valuing Equity to Valuing the Firm

A firm is more than just its equity investors. It has other claim holders, including bondholders and banks. When we value the firm, therefore, we consider cash flows to all of these claim holders. We define the **free cash flow to the firm** as being the cash flow left over after operating expenses, taxes and reinvestment needs, but before any debt payments (interest or principal payments).

Free Cash Flow to Firm (FCFF) = After-tax Operating Income – Reinvestment Needs

The two differences between FCFE and FCFF become clearer when we compare their definitions. The free cash flow to equity begins with net income, which is after interest expenses and taxes, whereas the free cash flow to the firm begins with after-tax operating income, which is before interest expenses. Another difference is that the FCFE is after net debt payments, whereas the FCFF is before net debt.

What exactly does the free cash flow to the firm measure? On the one hand, it measures the cash flows generated by the assets before any financing costs are considered and thus is a measure of operating cash flow. On the other, the free cash flow to the firm is the cash flow used to service all claim holders' needs for cash – interest and principal to debt holders and dividends and stock buybacks to equity investors.

To illustrate the estimation of free cash flow to the firm, consider Boeing in 1998. In that year, Boeing had adjusted operating income of \$ 2,736 million, a tax rate of 35% and reinvested \$1,719 million in new investments. The free cash flow to the firm for Boeing in 1998 is then:

$FCFF_{Boeing}$ = Operating Income (1- Tax Rate) – Reinvestment Needs

= \$ 2,736 (1-.35) - \$ 1,719 million = \$ 59 million

Once the free cash flows to the firm have been estimated, the process of computing value follows a familiar path. If valuing a firm or business with free cash flows growing at a constant rate forever, we can use the perpetual growth equation:

$$\text{Value of Firm with FCFF growing at constant rate} = \frac{E(FCFF_1)}{(k_c - g_n)}$$

There are two key distinctions between this model and the constant-growth FCFE model used earlier. The first is that we consider cash flows before debt payments in this model, whereas we used cash flows after debt payments when valuing equity. The second is that we then discount these cash flows back at a composite cost of financing, i.e., the cost of capital to arrive at the value of the firm, while we used the cost of equity as the discount rate when valuing equity.

To value firms where free cash flows to the firm are growing at a rate higher than that of the economy, we can modify this equation to consider the present value of the cash flows until the firm is in stable growth. To this present value, we add the present value of the terminal value, which captures all cash flows in stable growth.

$$\text{Value of high - growth business} = \sum_{t=1}^{t=N} \frac{E(FCFF_t)}{(1+k_c)^t} + \frac{\text{Terminal Value of Business}_N}{(1+k_c)^N}$$

Assume now that Boeing is interested in selling its information, space and defense systems division. The division reported cash flows before debt payments but after reinvestment needs of $ 393 million in 1998, and the cash flows are expected to grow 5% a year in the long term. The cost of capital for the division is 9%. The division can be valued as follows:

Value of Division = $ 393 (1.05) / (.09 - .05) = $ 10,318 million

fcffginzu.xls: See the spreadsheet that contains the valuation of Boeing as a firm.

You can extend this model to value Boeing as a firm. To do this valuation, assume that Boeing has cash flows before debt payments but after reinvestment needs and taxes of $ 850 million in the current year. Further, assume that these cash flows will grow at 15% a year for the next 5 years and at 5% thereafter. Boeing has a cost of capital of 9.17%. The value of Boeing as a firm can then be estimated in Table 4.7:

Table 4.7: Value of Boeing

Year	Cash Flow	Terminal Value	Present Value
1	$978		$895
2	$1,124		$943
3	$1,293		$994
4	$1,487		$1,047
5	$1,710	$43,049	$28,864
Value of Boeing as a firm =			$32,743

The terminal value is estimated using the free cash flow to the firm in year 6, the cost of capital of 9.17% and the expected constant growth rate of 5% as follows:

Terminal Value = $ 1710 (1.05)/(.0917-.05) = $ 43,049 million

It is then discounted back to the present to get the value of the firm today shown above as $32,743 million.

Note that this is not the value of the equity of the firm. To get to the value of the equity, we would need to subtract out debt from $32,743 million the value of all non-equity claims in the firm.

II. Relative Valuation

In intrinsic valuation the objective is to find assets that are priced below what they should be, given their cash flow, growth and risk characteristics. In relative valuation, the philosophical focus is on finding assets that are cheap or expensive relative to how "similar" assets are being priced by the market right now. It is therefore entirely possible that an asset that is expensive on an intrinsic value basis may be cheap on a relative basis.

A. Standardized Values and Multiples

To compare the valuations of "similar" assets in the market, we need to standardize the values in some way. They can be standardized relative to the earnings that they generate, the book value or replacement value of the assets themselves or relative to the revenues that they generate. Each approach is used widely and has strong adherents.

1. Earnings Multiples

One of the more intuitive ways to think of the value of any asset is as a multiple of the earnings generated by it. When buying a stock, it is common to look at the price paid as a multiple of the earnings per share generated by the company. This *price/earnings ratio* can be estimated using current earnings per share (which is called a trailing PE) or a expected earnings per share in the next year (called a forward PE). When buying a business (as opposed to just the equity in the business) it is common to examine the value of the business as a multiple of the operating income (or EBIT) or the operating cash flow (EBITDA). While a lower multiple is better than a higher one, these multiples will be affected by the growth potential and risk of the business being acquired.

2. Book Value or Replacement Value Multiples

While markets provide one estimate of the value of a business, accountants often provide a very different estimate of the same business in their books. This latter estimate, which is the *book value*, is driven by accounting rules and are heavily influenced by what was paid originally for the asset and any accounting adjustments (such as depreciation) made since. Investors often look at the relationship between the price they pay for a stock and the book value of equity (or net worth) as a measure of how over or undervalued a stock it; the price/book value ratio that emerges can vary widely across sectors, depending again upon the growth potential and the quality of the investments in each. When valuing businesses, this ratio is estimated using the value of the firm and the book value of all assets (rather than just the equity). For those who believe that book value is not a good measure of the true value of the assets, an alternative is to use the replacement cost of the assets; the ratio of the value of the firm to replacement cost is called *Tobin's Q*.

3. Revenue Multiples

Both earnings and book value are accounting measures and are affected by accounting rules and principles. An alternative approach, which is far less affected by these factors, is to look at the relationship between value of an asset and the revenues it generates. For equity investors, this ratio is the *price/sales ratio*, where the market value per share is divided by the revenues generated per share. For firm value, this ratio can be modified as the *value/sales ratio*, where the numerator becomes the total value of the firm. This ratio, again, varies widely across sectors, largely as a function of the profit margins in each. The advantage of these multiples, however, is that it becomes far easier to compare firms in different markets, with different accounting systems at work.

B. The Fundamentals Behind Multiples

One reason commonly given for relative valuation is that it requires far fewer assumptions than does discounted cash flow valuation. In my view, this is a misconception. The difference between discounted cash flow valuation and relative valuation is that the assumptions that an analyst makes have to be made explicit in the former and they can remain implicit in the latter. It is important that we know what the variables are that drive multiples, since these are the variables we have to control for when comparing these multiples across firms.

To look under the hood, so to speak, of equity and firm value multiples, we will go back to fairly simple discounted cash flow models for equity and firm value and use them to derive our multiples. Thus, the simplest discounted cash flow model for equity which is a stable growth dividend discount model would suggest that the value of equity is:

$$\text{Value of Equity} = P_0 = \frac{DPS_1}{k_e - g_n}$$

where DPS_1 is the expected dividend in the next year, k_e is the cost of equity and g_n is the expected stable growth rate. Dividing both sides by the earnings, we obtain the discounted cash flow model for the PE ratio for a stable growth firm:

$$\frac{P_0}{EPS_0} = PE = \frac{\text{Payout Ratio} * (1 + g_n)}{k_e - g_n}$$

Dividing both sides by the book value of equity, we can estimate the Price/Book Value ratio for a stable growth firm:

$$\frac{P_0}{BV_0} = PBV = \frac{\text{ROE} * \text{Payout Ratio} * (1 + g_n)}{k_e - g_n}$$

where ROE is the return on equity. Dividing by the Sales per share, the price/sales ratio for a stable growth firm can be estimated as a function of its profit margin, payout ratio, profit margin and expected growth.

$$\frac{P_0}{Sales_0} = PS = \frac{\text{Profit Margin} * \text{Payout Ratio} * (1 + g_n)}{k_e - g_n}$$

We can do a similar analysis from the perspective of firm valuation. The value of a firm in stable growth can be written as:

$$\text{Value of Firm} = V_0 = \frac{FCFF_1}{k_c - g_n}$$

Dividing both sides by the expected free cash flow to the firm yields the Value/FCFF multiple for a stable growth firm:

$$\frac{V_0}{FCFF_1} = \frac{1}{k_c - g_n}$$

Since the free cash flow the firm is the after-tax operating income netted against the net capital expenditures and working capital needs of the firm, the multiples of EBIT, after-tax EBIT and EBITDA can also be similarly estimated. The value/EBITDA multiple, for instance, can be written as follows:

$$\frac{\text{Value}}{\text{EBITDA}} = \frac{(1 - t)}{k_c - g} + \frac{\text{Depr (t)/EBITDA}}{k_c - g} - \frac{\text{CEx/EBITDA}}{k_c - g} - \frac{\Delta\text{ Working Capital/EBITDA}}{k_c - g}$$

The point of this analysis is not to suggest that we go back to using discounted cash flow valuation but to get a sense of the variables that may cause these multiples to vary across firms in the same sector. An analyst who is blind to these variables might conclude that a stock with a PE of 8 is cheaper than one with a PE of 12, when the true reason may by that the latter has higher expected growth, or that a stock with a P/BV ratio of 0.7 is cheaper than one with a P/BV ratio of 1.5, when the true reason may be that the latter has a much higher return on equity. The following table lists out the multiples that are widely used and the variables driving each; the variable, which in my view, is the most significant is highlighted for each multiple. This is what I would call the *companion variable* for this multiple, i.e., the one variable I would need to know in order to use this multiple to find under or over valued assets.

Table 4.8: Multiples and Companion Variables

Companion variables are in bold type

Multiple	*Determining Variables*
Price/Earnings Ratio	*Growth*, Payout, Risk
Price/Book Value Ratio	Growth, Payout, Risk, ***ROE***
Price/Sales Ratio	Growth, Payout, Risk, *Net Margin*
Value/EBIT Value/EBIT (1-t) Value/EBITDA	Growth, ***Reinvestment Needs***, Leverage, Risk
Value/Sales	Growth, Net Capital Expenditure needs, Leverage, Risk, ***Operating Margin***
Value/Book Capital	Growth, Leverage, Risk and ***ROC***

C. The Use of Comparables

Most analysts who use multiples use them in conjunction with "comparable" firms to form conclusions about whether firms are fairly valued or not. At the risk of being simplistic, the analysis begins with two decisions - the multiple that will be used in the analysis and the group of firms that will comprise the comparable firms. The multiple is computed for each of the comparable firms, and the average is computed. To evaluate an individual firm, the analyst then compares its multiple to the average computed; if it is significantly different, the analyst makes a subjective judgment on whether the firm's individual characteristics (growth, risk ..) may explain the difference. Thus, a firm may have a PE ratio of 22 in a sector where the average PE is only 15, but the analyst may conclude that this difference can be justified by the fact that the firm has higher growth potential than the average firm in the sector. If, in the analysts' judgment, the difference on the multiple cannot be explained by the fundamentals, the firm will be viewed as over valued (if its multiple is higher than the average) or undervalued (if its multiple is lower than the average).

1. Choosing Comparables

The heart of this process is the selection of the firms that comprise comparable firms. From a valuation perspective, a comparable firm is one with similar cash flows, growth potential and risk. If life were simple, the value of a firm would be analyzed by looking at how an exactly identical firm - in terms of risk, growth and cash flows - is priced. In most analyses, however, a comparable firm is defined to be one in the same business as the firm being analyzed. If there are enough firms in the sector to allow for it, this list will be pruned further using other criteria; for instance, only firms of similar size may be considered. Implicitly, the assumption being made here is that firms in the same sector have similar risk, growth and cash flow profiles and therefore can be compared with much more legitimacy. This approach becomes more difficult to apply under two conditions:

1. There are relatively few firms in a sector. In most markets outside the United States, the number of publicly traded firms in a particular sector, especially if it is defined narrowly, is small.
2. The differences on risk, growth and cash flow profiles across firms within a sector is large. Thus, there may be hundreds of computer software companies listed in the United States, but the differences across these firms are also large.

The tradeoff is therefore a simple one. Defining a sector more broadly increases the number of firms that enter the comparable firm list, but it also results in a more diverse group.

2. Controlling for Differences across Firms

Since it is impossible to find identical firms to the one being valued, we have to find ways of controlling for differences across firms on the relevant ways. The advantage of the discounted cash flow models introduced in the prior section is that we have a clear idea of what the fundamental determinants of each multiple are, and therefore what we should be controlling for; table 1 provides a summary of the variables. The process of controlling for the variables can range from very simple approaches, which modify the multiples to take into account differences on one key variable, to more complex approaches that allow for differences on more than one variable.

Let us start with the simple approaches. Here, the basic multiple is modified to take into account the most important variable determining that multiple. Thus, the PE ratio is divided by the expected growth rate in EPS for a company to come up with a growth-adjusted PE ratio. Similarly, the PBV ratio is divided by the ROE to come up with a value ratio, and the price sales ratio by the net margin. These modified ratios are then compared across companies in a sector. Implicitly, the assumption made is that these firms are comparable on all the other dimensions of value, besides the one being controlled for.

Illustration 4: Comparing PE ratios and growth rates across firms: Software companies

In the following table, we have listed the PE ratios and expected analyst consensus growth rates over 5 years for a selected list of software companies:

Company	*PE*	*Expected Growth Rate*	*PE/Expected Growth (PEG)*
Acclaim Entertainment	13.70	23.60%	0.58
Activision	75.20	40.00%	1.88
Broderbund	32.30	26.00%	1.24
Davidson Associates	44.30	33.80%	1.31
Edmark	88.70	37.50%	2.37
Electronic Arts	33.50	22.00%	1.52
The Learning Co.	33.50	28.80%	1.16
Maxis	73.20	30.00%	2.44
Minnesota Educational	69.20	28.30%	2.45
Sierra On-Line	43.80	32.00%	1.37

While comparisons on the PE ratio alone do not factor in the differences in expected growth, the PEG ratio in the last column can be viewed as growth adjusted PE ratio and that would suggest that Acclaim is the cheapest company in this group and Minnesota Educational is the most expensive. This conclusion holds only if these firms are of equivalent risk, however.

Controlling for more than one variable

When firms vary on more than one dimension, it becomes difficult to modify the multiples to take into account the differences across firms. It is, however, feasible to run regressions of the multiples against the variables and then use these regressions to get predicted values for each firm. This approach works reasonably well when the number of comparable firms is large and the relationship between the multiple and variable is strong. When these conditions do not hold, a few outliers can cause the coefficients to change dramatically and make the predictions much less reliable.

oilcos.xls: See the spreadsheet that contains the relative valuation of oil companies used in this example.

Illustration 5: PBV Ratios and ROE: The Oil Sector

The following table summarizes Price/Book Value ratios of oil companies and reports on their returns on equity and expected growth rates:

Company Name	*P/BV*	*ROE*	*Expected Growth*

Total ADR B	0.90	4.10	9.50%
Giant Industries	1.10	7.20	7.81%
Royal Dutch Petroleum ADR	1.10	12.30	5.50%
Tesoro Petroleum	1.10	5.20	8.00%
Petrobras	1.15	3.37	15%
YPF ADR	1.60	13.40	12.50%
Ashland	1.70	10.60	7%
Quaker State	1.70	4.40	17%
Coastal	1.80	9.40	12%
Elf Aquitaine ADR	1.90	6.20	12%
Holly	2.00	20.00	4%
Ultramar Diamond Shamrock	2.00	9.90	8%
Witco	2.00	10.40	14%
World Fuel Services	2.00	17.20	10%
Elcor	2.10	10.10	15%
Imperial Oil	2.20	8.60	16%
Repsol ADR	2.20	17.40	14%
Shell Transport & Trading ADR	2.40	10.50	10%
Amoco	2.60	17.30	6%
Phillips Petroleum	2.60	14.70	7.50%
ENI SpA ADR	2.80	18.30	10%
Mapco	2.80	16.20	12%
Texaco	2.90	15.70	12.50%
British Petroleum ADR	3.20	19.60	8%
Tosco	3.50	13.70	14%

Since these firms differ on both growth and return on equity, we ran a regression of PBV ratios on both variables:

PBV = -0.11 + 11.22 (ROE) + 7.87 (Expected Growth) $R^2 = 60.88\%$
(5.79) (2.83)

The numbers in brackets are t-statistics and suggest that the relationship between PBV ratios and both variables in the regression are statistically significant. The R-squared indicates the percentage of the differences in PBV ratios that is explained by the independent variables. Finally, the regression itself can be used to get predicted PBV ratios for the companies in the list. Thus, the predicted PBV ratio for Repsol would be:

Predicted PBV_{Repsol} = -0.11 + 11.22 (.1740) + 7.87 (.14) = 2.94
Since the actual PBV ratio for Repsol was 2.20, this would suggest that the stock was undervalued by roughly 25%.

Both approaches described above assume that the relationship between a multiple and the variables driving value are linear. Since this is not necessarily true, it is possible to run non-linear versions of these regressions.

3. Expanding the Comparable Firm Universe

Searching for comparable firms within the sector in which a firm operates is fairly restrictive, especially when there are relatively few firms in the sector or when a firm operates in more than one sector. Since the definition of a comparable firm is not one that is in the same business but one that has the same growth, risk and cash flow characteristics as the firm being analyzed, it is also unclear why we have to stay sector-specific. A software firm should be comparable to an automobile firm, if we can control for differences in the fundamentals.

The regression approach that we introduced in the previous section allows us to control for differences on those variables that we believe cause differences in multiples across firms. Using the minimalist version of the regression equations here, we should be able to regress PE, PBV and PS ratios against the variables that should affect them:

PE = a + b (Growth) + c (Payout ratios) + d (Risk)
PBV = a + b (Growth) + c (Payout ratios) + d (Risk) + e (ROE)
PS = a + b (Growth) + c (Payout ratios) + d (Risk) + e (Margin)

It is, however, possible that the proxies that we use for risk (beta) , growth (expected growth rate) and cash flow (payout) may be imperfect and that the relationship may not be linear. To deal with these limitations, we can add more variables to the regression - e.g., the size of the firm may operate as a good proxy for risk - and use transformations of the variables to allow for non-linear relationships.

The first advantage of this approach over the “subjective” comparison across firms in the same sector described in the previous section is that it does quantify, based upon actual market data, the degree to which higher growth or risk should affect the multiples. It is true that these estimates can be noisy, but this noise is a reflection of the reality that many analysts choose not to face when they make subjective judgments. Second, by looking at all firms in the universe, it allows analysts operating in sectors with relatively few firms in them to make more powerful comparisons. Finally, it gets analysts past the tunnel vision induced by comparing firms within a sector, when the entire sector may be under or over valued.

Valuing an Asset with Contingent Cash Flows (Options)

In general, the value of any asset is the present value of the expected cash flows on that asset. In this section, we will consider an exception to that rule when we will look at assets with two specific characteristics:

- They derive their value from the values of other assets.
- The cash flows on the assets are contingent on the occurrence of specific events.

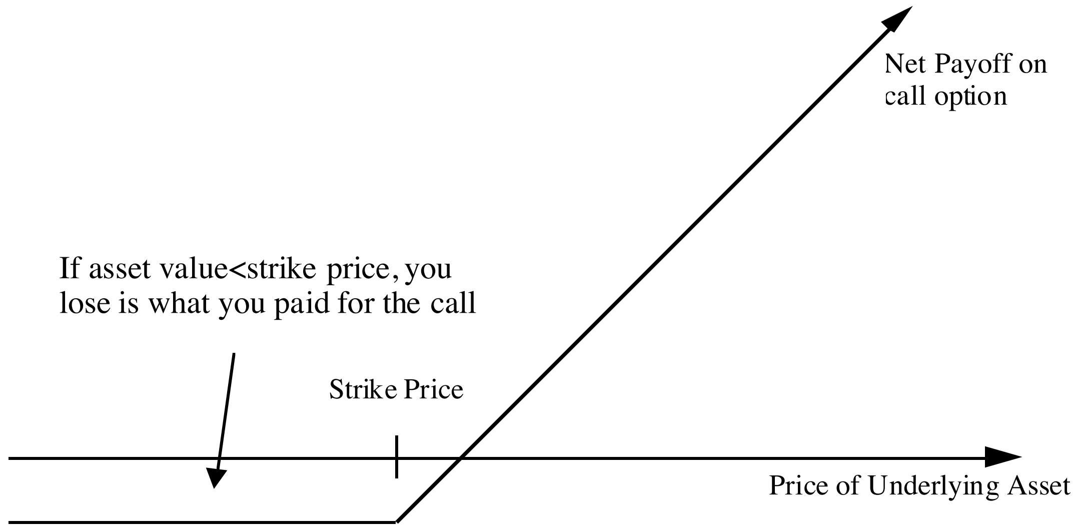
These assets are called options, and the present value of the expected cash flows on these assets will understate their true value. In this section, we will describe the cash flow characteristics of options, consider the factors that determine their value and examine how best to value them.

Cash Flows on Options

There are two types of options. A call option gives the buyer of the option the right to buy the underlying asset at a fixed price, whereas a put option gives the buyer the right to sell the underlying asset at a fixed price. In both cases, the fixed price at which the underlying asset can be bought or sold is called the **strike or exercise price**.

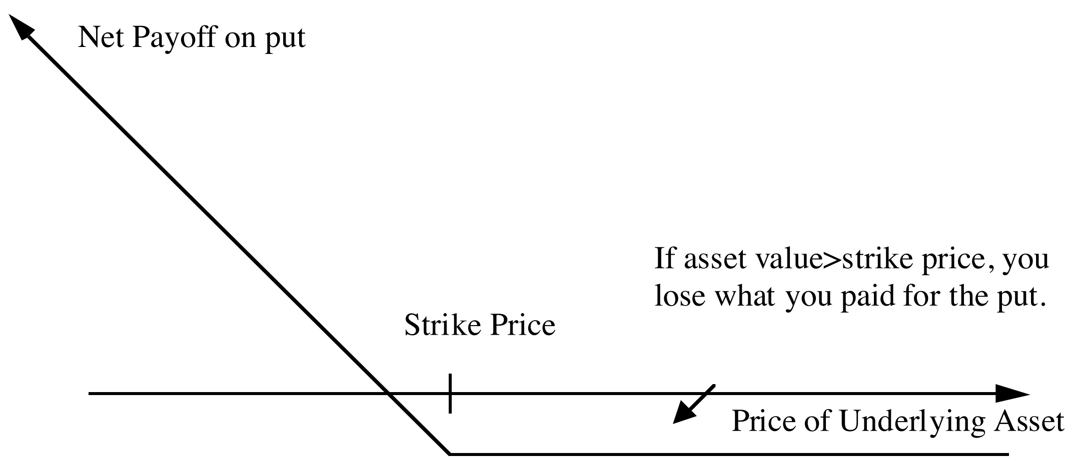
To look at the payoffs on an option, consider first the case of a call option. When you buy the right to sell an asset at a fixed price, you want the price of the asset to increase above that fixed price. If it does, you make a profit, since you can buy at the fixed price and then sell at the much higher price; this profit has to be netted against the cost initially paid for the option. However, if the price of the asset decreases below the strike price, it does not make sense to exercise your right to buy the asset at a higher price. In this scenario, you lose what you originally paid for the option. Figure 4.8 summarizes the cash payoff at expiration to the buyer of a call option.

Figure 4.8: Payoff on Call Option



With a put option, you get the right to sell at a fixed price, and you want the price of the asset to decrease below the exercise price. If it does, you buy the asset at the exercise price and then sell it back at the current price, claiming the difference as a gross profit. When the initial cost of buying the option is netted against the gross profit, you arrive at an estimate of the net profit. If the value of the asset rises above the exercise price, you will not exercise the right to sell at a lower price. Instead, the option will be allowed to expire without being exercised, resulting in a net loss of the original price paid for the put option. Figure 4.9 summarizes the net payoff on buying a put option.

Figure 4.9: Payoff on Put Option



With both call and put options, the potential for profit to the buyer is significant, but the potential for loss is limited to the price paid for the option.

Determinants of Option Value

What is it that determines the value of an option? At one level, options have expected cash flows just like all other assets, and that may seem like good candidates for discounted cash flow valuation. The two key characteristics of options -- that they derive their value from some other traded asset, and the fact that their cash flows are contingent on the occurrence of a specific event -- does suggest an easier alternative. We can create a portfolio that has the same cash flows as the option being valued, by combining a position in the underlying asset with borrowing or lending. This portfolio is called a **replicating portfolio** and should cost the same amount as the option. The principle that two assets (the option and the replicating portfolio) with identical cash flows cannot sell at different prices is called the **arbitrage principle**.

Options are assets that derive value from an underlying asset; increases in the value of the underlying asset will increase the value of the right to buy at a fixed price and reduce the value to sell that asset at a fixed price. On the other hand, increasing the strike price will reduce the value of calls and increase the value of puts.

While calls and puts move in opposite directions when stock prices and strike prices are varied, they both increase in value as the life of the option and the variance in the underlying asset's value increases. The reason for this is the fact that options have limited losses. Unlike traditional assets that tend to get less valuable as risk is increased, options become more valuable as the underlying asset becomes more volatile. This is so because the added variance cannot worsen the downside risk (you still cannot lose more than what you paid for the option) while making potential profits much higher. In addition, a longer life for the options just allows more time for both call and put options to appreciate in value. Since calls provide the right to buy the underlying asset at a fixed price, an increase in the value of the asset will increase the value of the calls. Puts, on the other hand, become less valuable as the value of the asset increase.

The final two inputs that affect the value of the call and put options are the riskless interest rate and the expected dividends on the underlying asset. The buyers of call and put options usually pay the price of the option up front, and wait for the expiration day to exercise. There is a present value effect associated with the fact that the promise to buy an asset for $ 1 million in 10 years is less onerous than paying it now. Thus, higher interest rates will generally increase the value of call options (by reducing the present value of the price on exercise) and decrease the value of put options (by decreasing the present value ofthe price received on exercise). The expected dividends paid by assets make them less valuable; thus, the call option on a stock that does not pay a dividend should be worth more than a call option on a stock that does pay a dividend. The reverse should be true for put options.

Conclusion

In this chapter, we lay the foundations for the models that we will be using to value both assets and firms in the coming chapters. There are three classes of valuation models. The more general of these models, discounted cash flow valuation, can be used to value any asset with expected cash flows over its life. The value is the present value of the expected cash flows at a discount rate that reflects the riskiness of the cash flows, and this principle applies whether one is looking at a zero-coupon government bond or equity in high risk firms. The second set of models are relative valuation models, where we value assets based upon how similar assets are priced by the market. There are some assets that generate cash flows only in the event of a specified contingency, and these assets will not be valued accurately using discounted cash flow models. Instead, they should be viewed as options and valued using option pricing models.

Lessons for Investors

1. All assets that generate or are expected to generate cashflows can be valued by discounting the expected cash flows back at a rate that reflects the riskiness of the cashflows – more risky cash flows should be discounted at higher rates.
2. The value of an on-going business is a function of four variables – how much the business generates in cashflows from existing investments, how long these cashflows can be expected to grow at a rate higher than the growth rate of the economy (high growth period), the level of the growth rate during this period and the riskiness of the cashflows. Companies with higher cashflows, higher growth rates, longer high-growth periods and lower risk will have higher values.
3. Alternatively, assets can be valued by looking at how similar assets are priced in the market. This approach is called relative valuation and is built on the presumption that the market is correct, on average.
4. Assets whose cashflows are contingent on the occurrence of specific events are called options and can be valued using option pricing models.